Second Order Sliding Mode Observer for Mechanical Systems with Coulomb Friction

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Resumen

The super-twisting second-order sliding-mode algorithm is modified in order to design a velocity observer for systems with Coulomb friction. The finite time convergence of the observer is proved. A discrete version of the observer is considered and the corresponding accuracy is estimated.

1. INTRODUCTION

The design of observers for the mechanical systems with Coulomb friction is important for the following reasons:

- linear observers do not achieve adequate performance for such systems;
- model based observers are usually restricted to cases when the model is exactly known;
- high-gain differentiators [2] are not exact with any fixed finite gain and feature the peaking effect with high gains: the maximal output value during the transient grows infinitely as the gains tend to infinity (see, for example,[5],[11],[3]).

The sliding mode observers are widely used due to the finite-time convergence, robustness with respect to uncertainties and the possibility of uncertainty estimation (see, for example, the bibliography in the recent tutorials [5],[11],[3]). New generation of observers based on the second-order sliding-mode algorithms has been recently developed and used:

- Observers with asymptotic convergence of error were developed in [12] based on second order sliding modes.
- In [1] an asymptotic observer for systems with Coulomb friction was designed based on the second-order sliding-mode but this observer required the proof of separation principle theorem.

For above mentioned observers the proof of separation principle theorem was necessary due to the asymptotic convergence of the estimated values to the real ones.
In [9] a robust exact differentiator ensuring finite time convergence was designed, as an application of the super-twisting algorithm [8]. These differentiators were, for example, successfully applied in [13], [4], [10]. A new differentiator [7] was developed, based on it.
If the mathematical model of controlled system is known, or the parameters and uncertainties of model can be estimated (which is common for the case of mechanical systems with Coulomb friction), it is reasonable to design a velocity observer.
In this paper we design an observer which reconstructs the velocity from position measurements basing on the second order sliding modes super-twisting algorithm [8] with the finite time convergence. The designed observer allows to design the controller and observer separately and consequently does not require the separation principle theorem. The constants of the observer allowing to take into account the partial knowledge of the systems model and for example omit the elasticity therm. The discrete version of super twisting algorithm is firstly considered and for the proposed observer the corresponding accuracy is estimated and proved.

2. PROBLEM STATEMENT

The general model of second order mechanical systems has the form

$$M(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}} + P(\dot{\mathbf{q}}) + G(\mathbf{q}) + \Delta(t,\mathbf{q},\dot{\mathbf{q}}) = \tau, \quad (1)$$

where $\mathbf{q} \in R^n$ is a vector of generalized coordinates,$M(\mathbf{q})$ is the inertia matrix, $C(\mathbf{q},\dot{\mathbf{q}})\dot{\mathbf{q}}$ the

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term of Coriolis and centrifugal forces, $P(\dot{\mathbf{q}})$ is the Coulomb friction which could consists on relay term depends on $\dot{\mathbf{q}}$, $G(\mathbf{q})$ is the term of gravitational forces, $\Delta(t, \mathbf{q}, \dot{\mathbf{q}})$ is an uncertainty term and τ is the torque produced by the actuators. Notice the matrix $M(\mathbf{q}) = M^T(\mathbf{q})$ is strictly positive definite, then $M^{-1}(\mathbf{q})$ does exist.
Introducing the variables $\mathbf{x}_1 = \mathbf{q}, \mathbf{x}_2 = \dot{\mathbf{q}}, u = \tau$, the model (1) can be rewritten in the state space form as:

$$\begin{aligned} \dot{\mathbf{x}}_1 &= \mathbf{x}_2, \\ \dot{\mathbf{x}}_2 &= f(t, \mathbf{x}_1, \mathbf{x}_2, u) + \xi(t, \mathbf{x}_1, \mathbf{x}_2), \\ \mathbf{y} &= \mathbf{x}_1, \end{aligned} \tag{2}$$

where $f(t, \mathbf{x}_1, \mathbf{x}_2, u) = -M^{-1}(\mathbf{x}_1)[C(\mathbf{x}_1, x_2)\mathbf{x}_2 + G(\mathbf{x}_1) - u]$ and $\xi(t, \mathbf{x}_1, \mathbf{x}_2) = -M^{-1}(\mathbf{x}_1)(P(\mathbf{x}_2) + \Delta(t, \mathbf{x}_1, \mathbf{x}_2))$. Suppose that ξ and u are bounded and that the system (2) has a right-hand unique solution in Filippov's sense [6].
The task is to design an observer of the velocity ($\frac{d\mathbf{q}}{dt} = \dot{\mathbf{q}}$) with finite time convergence for the original system (1), which for the system (2) is given by the state $\mathbf{x}_2$, with the only measurement of the position ($\mathbf{q}$), i.e. the state x_1.

3. OBSERVER DESIGN

The proposed super-twisting observer has the form

$$\begin{aligned} \dot{\hat{\mathbf{x}}}_1 &= \hat{\mathbf{x}}_2 + \mathbf{z}_1 \\ \dot{\hat{\mathbf{x}}}_2 &= f(t, \mathbf{x}_1, \hat{\mathbf{x}}_2, u) + \mathbf{z}_2 \end{aligned} \tag{3}$$

where $\hat{\mathbf{x}}_1$ and $\hat{\mathbf{x}}_2$ are the state estimations, $\mathbf{z}_1$ and $\mathbf{z}_2$ are the correction factors based on the super-twisting algorithm are given by the formulas

$$\begin{cases} z_{1j} = \lambda |x_{1j} - \hat{x}_{1j}|^{1/2}\, sign(x_{1j} - \hat{x}_{1j}) \\ z_{2j} = \alpha\, sign(x_{1j} - \hat{x}_{1j}), .j = 1, .., n \end{cases} \tag{4}$$

where z_{1j}, z_{2j}, x_{1j} and $\hat{x}_{1j}$ are corresponding coordinates of the vectors $\mathbf{z}_1$,$\mathbf{z}_2$, $\mathbf{x}_1$ and $\hat{\mathbf{x}}_1$ respectively. Assume that

$$||f(t, \mathbf{x}_1, \mathbf{x}_2, u) - f(t, \mathbf{x}_1, \hat{\mathbf{x}}_2, u) + \xi(t, \mathbf{x}_1, \mathbf{x}_2)|| \leq f^+.$$

Note that the estimation constant f^+ does not depend on the elasticity terms. Let α and λ satisfy the inequalities

$$\begin{aligned} \alpha &> f^+ \\ \lambda &> \sqrt{\tfrac{2}{\alpha - f^+} \tfrac{(\alpha + f^+)(1+q)}{(1-q)}} \end{aligned} \tag{5}$$

where q is some chosen constant, $0 < q < 1$.
Taking $\tilde{\mathbf{x}}_1 = \mathbf{x}_1 - \hat{\mathbf{x}}_1$ and $\tilde{\mathbf{x}}_2 = \mathbf{x}_2 - \hat{\mathbf{x}}_2$, $g(t, \mathbf{x_1}, \mathbf{x}_2, \hat{\mathbf{x}}_2, u) = f(t, \mathbf{x_1}, \mathbf{x}_2, u) - f(t, \mathbf{x_1}, \hat{\mathbf{x}}_\mathbf{2}, u) + \xi(t, \mathbf{x_1}, \mathbf{x_2})$ we obtain the equations for the error in coordinates in the following form:

$$\begin{aligned} \dot{\tilde{x}}_{1j} &= \tilde{x}_{2j} - \lambda |\tilde{x}_{1j}|^{1/2}\, sign(\tilde{x}_{1j}) \\ \dot{\tilde{x}}_{2j} &= g_j(t, \mathbf{x}_1, \mathbf{x}_2, \hat{\mathbf{x}}_2, u) - \alpha\, sign(\tilde{x}_{1j}), j = 1, .., .n \end{aligned} \tag{6}$$

where $g_j(t, \mathbf{x}_1, \mathbf{x}_2, \hat{\mathbf{x}}_2, u)$ is a corresponding coordinate of the function g.

Theorem 1 *The observer (3),(4) for the system (2) ensures the finite time convergence of estimated states to the real states, i.e.* $(\hat{\mathbf{x}}_1, \hat{\mathbf{x}}_2) \to (\mathbf{x}_1, \mathbf{x}_2)$.

To prove the convergence of the state estimates to the real states, it is necessary to prove first the convergence of $\tilde{\mathbf{x}}_1$ and $\dot{\tilde{\mathbf{x}}}_1$ to zero. Obviously, $\tilde{x}_{1j}$ and $\tilde{x}_{2j}$ for all $j = 1, ..., n$ satisfy the differential inclusion

$$\begin{aligned} \dot{\tilde{x}}_{1j} &= \tilde{x}_{2j} - \lambda |\tilde{x}_{1j}|^{1/2}\, sign(\tilde{x}_{1j}) \\ \dot{\tilde{x}}_{2j} &\in [-f^+, f^+] - \alpha\, sign(\tilde{x}_{1j}) \end{aligned} \tag{7}$$

Here and further all differential inclusions are understood in the Filippov sense, which means that the right hand side is enlarged in some points in order to satisfy the upper semicontinuity property [6], in particular the second formula of (7) turns into $\dot{\tilde{x}}_{2j} \in [-\alpha - f^+, \alpha + f^+]$ with $\tilde{x}_{1j} = 0$. Taking into account that all coordinates of the estimated states $\tilde{x}_{1j}$ and $\tilde{x}_{2j}$ for all $j = 1, ..., n$ satisfy to the same inclusions we will omit bellow in the proof of theorem 1 the index j meaning that $\tilde{x}_1$ and $\tilde{x}_2$ are arbitrary coordinates of the vectors $\tilde{\mathbf{x}}_1$ and $\tilde{\mathbf{x}}_2$ respectively.
Computing the derivative of $\dot{\tilde{x}}_1$ obtain

$$\ddot{\tilde{x}}_1 \in [-f^+, f^+] - (\frac{1}{2}\lambda \frac{\dot{\tilde{x}}_1}{|\tilde{x}_1|^{1/2}} + \alpha\, sign\, \tilde{x}_1) \tag{8}$$

The trivial identity $\frac{d}{dt}|x| = \dot{x}\, sign\, x$ is used here. Inclusion (8) does not "remember" anything on the real system, but can be used to describe the majorant curve drawn in the Fig. 1. In the case when $\tilde{x}_1 > 0$ and $\dot{\tilde{x}}_1 > 0$ the trajectory is confined between the axis $\tilde{x}_1 = 0$, $\dot{\tilde{x}}_1 = 0$ and the trajectory of the equation $\ddot{\tilde{x}}_1 = -(\alpha - f^+)$. Let $\tilde{x}_{1_M}$ be the intersection of this curve with the axis $\dot{\tilde{x}}_1 = 0$. Obviously, $2(\alpha - f^+)\tilde{x}_{1_M} = \dot{\tilde{x}}_{1_0}^2$. It is easy to see that for $\tilde{x}_1 > 0, \dot{\tilde{x}}_1 > 0$

$$\ddot{\tilde{x}}_1 \leq f^+ - \alpha\, sign\, \tilde{x}_1 - \frac{1}{2}\lambda \frac{\dot{\tilde{x}}_1}{|\tilde{x}_1|^{1/2}} < 0.$$

Thus the trajectory approaches the axis $\dot{\tilde{x}}_1 = 0$.
The majorant curve for $\tilde{x}_1 > 0, \dot{\tilde{x}}_1 \geq 0$ is described by the equation (see Fig. 1)

$$\dot{\tilde{x}}_1^2 = 2(\alpha - f^+)(\tilde{x}_{1_M} - \tilde{x}_1) \quad \text{for} \quad \dot{\tilde{x}}_1 > 0$$

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The majorant curve for $\tilde{x}_1 > 0, \dot{\tilde{x}}_1 \le 0$ consists of two parts. In the first part the point instantly drops down from $(\tilde{x}_{1_M}, 0)$ to the point $(\tilde{x}_{1_M}, -\frac{2}{\lambda}(f^+ + \alpha)\tilde{x}_{1_M}^{1/2})$, where the right hand side of inclusion (8) in the 'worst case' is equal to zero. The second part of majorant curve is the horizontal segment between the points $(\tilde{x}_{1_M}, -\frac{2}{\lambda}(f^+ + \alpha)\tilde{x}_{1_M}^{1/2}) = (\tilde{x}_{1_M}, \dot{\tilde{x}}_{1_M})$ and $(0, \dot{\tilde{x}}_{1_M})$.

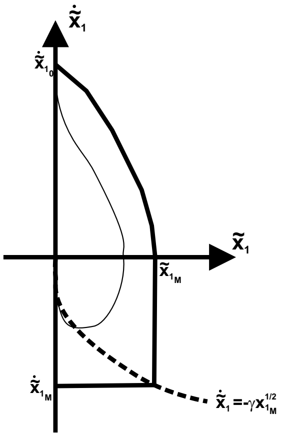

Figura 1: Majorant curve

Condition (5) implies that

$$\frac{|\dot{\tilde{x}}_{1_M}|}{|\dot{\tilde{x}}_{1_0}|} < \frac{1-q}{1+q} < 1.$$

Last inequality ensures the convergence of the state $(\tilde{x}_{1_i}, \dot{\tilde{x}}_{1_k})$ to $\tilde{x}_1 = \dot{\tilde{x}}_1 = 0$ and, moreover, the convergence of $\Sigma_0^\infty |\dot{\tilde{x}}_{1_k}|$. To prove the finite time of convergence consider the dynamics of $\tilde{x}_2$. Obviously, $\tilde{x}_2 = \dot{\tilde{x}}_1$ at the moments when $\tilde{x}_1 = 0$ and taking into account that

$$\dot{\tilde{x}}_{2j} = g_j(\mathbf{x}_1\mathbf{x}_2, \hat{\mathbf{x}}_2, u) - \alpha\, sign\, \tilde{x}_{1j}, j = 1, ..., n$$

(g_j is corresponding coordinate of vector g) obtain that

$$0 < \alpha - f^+ \le |\dot{\tilde{x}}_2| \le \alpha + f^+$$

holds in a small vicinity of the origin. Thus

$$|\dot{\tilde{x}}_{1k}| \ge (\alpha - f^+)t_k,$$

where t_k is the time interval between the successive intersection of the trajectory with the axis $\tilde{x}_1 = 0$. Hence

$$t_k \le \frac{|\dot{\tilde{x}}_{1_k}|}{(\alpha - f^+)}$$

and the total convergence time is given by

$$T \le \sum \frac{|\dot{\tilde{x}}_{1_k}|}{(\alpha - f^+)}$$

So T is finite and the estimated states converge to the real states in finite time.

Remark 1. The finite time convergence of observer allows to design the observer and the control law separately, i.e. the separation principle is satisfied. The only requirement for its implementation is the boundedness of the function $g(t, \mathbf{x}_1, \mathbf{x}_2, \hat{\mathbf{x}}_2, u)$.

Remark 2. The standard 2-sliding-mode-based differentiator can be locally applied for differentiation of each coordinate too. Selecting the observer's gain we can use the partial knowledge of system's model omiting the elasticity term $M^{-1}(\mathbf{x}_1)G(\mathbf{x}_1)$.

3.1. Discrete case

The above analysis is valid for the ideal version of the observer. Let $f, \mathbf{x}, \mathbf{z}_1, \mathbf{z_2}$ be measured at discrete times with the time interval δ, and let t_i, t_{i+1}be successive measurement times. Consider a discrete modification of the observer (the Euler scheme)

$$\begin{gathered}\hat{x}_{1j}(t_{i+1}) = \hat{x}_{1j}(t_i) + (\hat{x}_{2j}(t_i) + \lambda|x_{1j}(t_i) \\ -\hat{x}_{1j}(t_i)|^{1/2}\, sign(x_{1j}(t_i) - \hat{x}_{1j}(t_i)))\delta \\ \hat{x}_{2j}(t_{i+1}) = \hat{x}_{2j}(t_i) + (f_j(t_i, \mathbf{x}_1(t_i), \hat{\mathbf{x}}_2(t_i), u(t_i)) \\ +\alpha\, sign(x_{1j}(t_i) - \hat{x}_{1j}(t_i)))\delta, j = 1, .., .n\end{gathered}$$

where $\hat{x}_{1j}(t_i)$, $\hat{x}_{2j}(t_i)$ and f_j are the coordinates of the vectors $\hat{\mathbf{x}}_1(t_i)$, $\hat{\mathbf{x}}_2(t_i)$ and f respectively, $t_{i+1} - t_i = \delta$.

Theorem 2 *Suppose that the function f is uniformly bounded, then the observation algorithm (9) ensures the convergence of the estimation errors to the bounded region $||\tilde{\mathbf{x}}_1|| \le \gamma_1\delta^2$, $||\tilde{\mathbf{x}}_2|| \le \gamma_2\delta$ with some constants γ_1, γ_2, depending on the observer parameters.*

The observer (9) may be rewritten in the continuous time as follows:

$$\begin{gathered}\dot{\hat{x}}_{1j} = \hat{x}_{2j}(t_i) + \lambda|x_1(t_i) - \hat{x}_{1j}(t_i)|^{1/2}\, sign(x_{1j}(t_i) \\ -\hat{x}_{1j}(t_i)) \\ \dot{\hat{x}}_{2j} = f_j(t_i, \mathbf{x}_1(t_i), \hat{\mathbf{x}}_2(t_i), u(t_i)) \\ +\alpha\, sign(x_{1j}(t_i) - \hat{x}_{1j}(t_i)), j = 1, .., .n\end{gathered}$$

Hence, the errors satisfy the differential inclusion

$$\begin{gathered}\dot{\tilde{x}}_{1j} = \tilde{x}_{2j}(t_i) + x_{2j} - x_{2j}(t_i) \\ -\lambda|\tilde{x}_{1j}(t_i)|^{1/2}\, sign(\tilde{x}_{1j}(t_i)) \\ \dot{\tilde{x}}_{2j} \in [-f^+, f^+] - \alpha\, sign(\tilde{x}_{1j}(t_i)), j = 1, .., .n\end{gathered}$$

Taking into account that all coordinates of the estimated states $\tilde{\mathbf{x}}_1(t_i)$ and $\tilde{\mathbf{x}}_2(t_i)$ satisfy to the same inclusions, to simplify the notations, we will omit bellow in the proof of the theorem 2 the index j as the corresponding coordinate numbers meaning that $\tilde{x}_1$

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and $\tilde{x}_2$ are the arbitrary coordinates of the vectors $\tilde{\mathbf{x}}_1$.and $\tilde{\mathbf{x}}_2$ respectively.
Let $||f+\xi|| \leq f_1^+$, then

$$\begin{aligned} \dot{\tilde{x}}_1 &\in \tilde{x}_2(t_i) + [-f_1^+, f_1^+]\delta - \lambda|\tilde{x}_1(t_i)|^{1/2}\, sign(\tilde{x}_1(t_i)) \\ \dot{\tilde{x}}_2 &\in [-f^+, f^+] - \alpha\, sign(\tilde{x}_1(t_i)) \end{aligned} \tag{9}$$

It may be considered as (7) with measurement errors. Indeed, let D be some compact region around the origin O of the space $\tilde{x}_1, \tilde{x}_2$. As follows from the proof of Theorem 1, all trajectories of (7) starting in D converge in some finite time T to the origin O. During this time they do not leave some larger homogeneous disk $B_{R_0} = \{(\tilde{x}_1, \tilde{x}_2) \| \tilde{x}_1|^{1/2} + |\tilde{x}_2| \leq R_0\}$. Let $M(R) = \text{máx}\{|\tilde{x}_2 - \lambda|\tilde{x}_1|^{1/2}\, sign(\tilde{x}_1)| \mid (\tilde{x}_1, \tilde{x}_2) \in B_R\}$. Due to the homogeneity property $M(R) = mR$, where the constant $m > 0$ can be easily calculated. Thus, obviously, in B_{R_0}

$$|\tilde{x}_1(t) - \tilde{x}_1(t_i)| \leq mR_0\delta, \quad |\tilde{x}_2(t) - \tilde{x}_2(t_i)| \leq (f^+ + \alpha)\delta,$$

and, denoting $f_2^+ = f^+ + f_1^+ + \alpha$, obtain that the trajectories of (9) satisfy the inclusion

$$\begin{aligned} \dot{\tilde{x}}_1 &\in \tilde{x}_2 + [-f_2^+, f_2^+]\delta - \lambda|\tilde{x}_1 + ... \\ &+[-2m, 2m]R_0\delta|^{1/2}\, sign(\tilde{x}_1 + [-2m, 2m]R_0\delta) \\ \dot{\tilde{x}}_2 &\in [-f^+, f^+] - \alpha\, sign(\tilde{x}_1 + [-2m, 2m]R_0\delta) \end{aligned}$$

while $(\tilde{x}_1, \tilde{x}_2) \in B_{2R_0}$.With δ being zero, the dynamics (10) coincides with (7), whose trajectories converge in finite time to the origin. Due to the continuous dependence of the Filippov solutions on the graph of the differential inclusion, with sufficiently small δ the trajectories of (10) starting in D terminate in the time T in some small compact vicinity $\tilde{D} \subset D$ of the origin without leaving B_{2R_0}on the way. Let Ω be the compact set [6] of all points belonging to the trajectory segments starting in $\tilde{D}$ and corresponding to the closed time interval T, $\tilde{D} \subset \Omega$. With δ small enough $\tilde{D} \subset \Omega \subset D$, since the origin O is invariant for (10).
Obviously, Ω is an invariant set attracting the trajectories of (9) starting in D. Check now that it is a globally attracting set. Define a homogeneous parameter-time-coordinate transformation

$$\begin{aligned} t \longmapsto \eta t, \quad (\tilde{x}_1, \tilde{x}_2) &\longmapsto (\eta^2\tilde{x}_1, \eta\tilde{x}_2), \\ (R_0, \delta) &\longmapsto (\eta R_0, \eta\delta) \end{aligned} \tag{10}$$

and let $G_\eta(\tilde{x}_1, \tilde{x}_2) = (\eta^2\tilde{x}_1, \eta\tilde{x}_2)$. It is easily seen that (10) preserves (10), i.e. the trajectories are preserved. Choose such $\eta > 1$ that $G_\eta\Omega \subset D$, then the trajectories of the inclusion

$$\begin{aligned} \dot{\tilde{x}}_1 &\in \tilde{x}_2 + [-f_2^+, f_2^+]\eta\delta - \lambda|\tilde{x}_1 + ... \\ &+[-2m, 2m]R_0\eta^2\delta|^{1/2}\, sign(\tilde{x}_1 + [-2m, 2m]R_0\eta^2\delta) \\ \dot{\tilde{x}}_2 &\in [-f^+, f^+] - \alpha\, sign(\tilde{x}_1 + [-2m, 2m]R_0\eta^2\delta) \end{aligned}$$

starting in $G_\eta D$ terminate following time ηT in $G_\eta\Omega \subset D$ without leaving $G_\eta B_{2R_0} = B_{2\eta R_0}$ on the way. Comparing (10) and (11) obtain that (11) describes the solutions of (9) in $B_{2\eta R_0}$, but with redundantly enlarged "noise level"due to the replacement of δ by $\eta\delta > \delta$. Hence, the solutions of (9) satisfy (11) in $B_{2\eta R_0}$. Therefore, the trajectories of (9) starting in $G_\eta D$ terminate following time ηT in $G_\eta\Omega \subset D$ and proceed into Ω in time T. Representing the whole plane $\tilde{x}_1, \tilde{x}_2$ as $\mathbf{R}^2 = G_\eta^k D$ obtain the global finite-time convergence to the set Ω.
Let Ω satisfy the inequalities $|\tilde{x}_1| \leq a_1$, $|\tilde{x}_2| \leq a_2$ with some discretization interval δ_0. It is easy to see that (9) is invariant with respect to the transformation $t \longmapsto \eta t$, $(\tilde{x}_1, \tilde{x}_2) \longmapsto (\eta^2\tilde{x}_1, \eta\tilde{x}_2)$, $\delta \longmapsto \eta\delta$. Let Ω satisfy the inequalities $|\tilde{x}_1| \leq a_1$, $|\tilde{x}_2| \leq a_2$ with some discretization interval δ_0. Applying the transformation with $\eta = \delta/\delta_0$ obtain that with arbitrary $\delta > 0$ the invariant set satisfies the inequalities $|\tilde{x}_1| \leq \gamma_1\delta^2$, $|\tilde{x}_2| \leq \gamma_2\delta$ with $\gamma_1 = a_1/\delta_0^2$, $\gamma_2 = a_2/\delta_0$.

4. EXAMPLE

Consider a pendulum system with Coulomb friction given by the equation

$$\ddot{\theta} = \frac{1}{J}\tau - \frac{MgL}{2J}\sin\theta - \frac{V_s}{J}\dot{\theta} - \frac{P_s}{J}\, sign(\dot{\theta}),$$

where the values $M = 0{,}2$, $g = 9{,}81$, $L = 0{,}3$, $J = 0{,}05$, $V_S = 0{,}2$, $P_s = 0{,}25$ were taken. Let it be driven by the twisting controller

$$\tau = -2\, sign(\theta - \theta_d) - sign(\dot{\theta} - \dot{\theta}_d),$$

where $\theta_d = \sin t$ and $\dot{\theta}_d = \cos t$ are the reference signals. The system can be rewritten as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= \tfrac{1}{J}\tau - \tfrac{MgL}{2J}\sin x_1 - \tfrac{V_s}{J}x_2 - \tfrac{P_s}{J}\, sign(x_2) \end{aligned}$$

The calculation shows that $|-\frac{V_s}{J}\tilde{x}_2 - \frac{P_s}{J}\, sign(x_2)| \leq f^+ = 17$. Therefore, the observer parameters $\alpha = 20$ and $\lambda = 17$ were chosen. Note that the term $|\frac{MgL}{2J}\sin x_1|$ would be taken into account for the choice of the differentiator parameters [9]. Thus, the proposed velocity observer has the form

$$\begin{aligned} \dot{\hat{x}}_1 &= \hat{x}_2 + 17|\tilde{x}_1|^{1/2}\, sign(\tilde{x}_1) \\ \dot{\hat{x}}_2 &= \tfrac{1}{J}\tau - \tfrac{MgL}{2J}\sin x_1 - \tfrac{V_s}{J}\hat{x}_2 + 20\, sign(\tilde{x}_1) \end{aligned}$$

The initial values $\theta = x_1 = \pi/4$ and $\dot{\theta} = x_2 = 0$ were taken at $t = 0$. The performance of the observer with the sampling interval $\delta = 0{,}00001$ is shown in Fig. 2, the finite-time convergence of the estimated velocity to the real one is demonstrated in Fig. 3.

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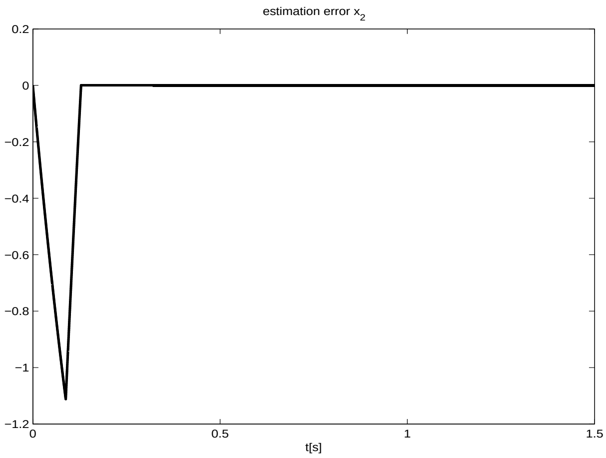

Figura 2: Estimation error for x_2.

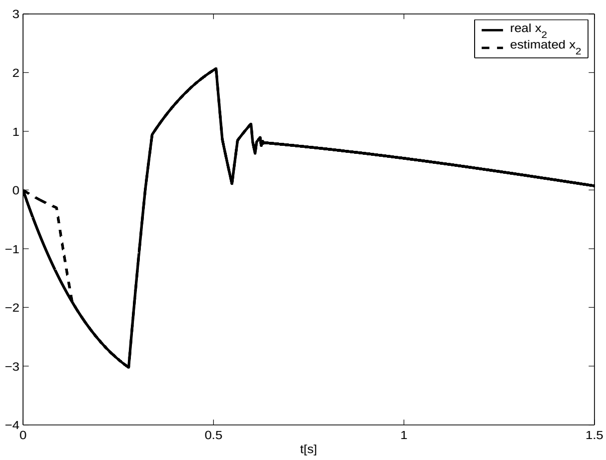

Figura 3: Real and estimated velocity.

5. CONCLUSIONS

The super-twisting second-order sliding-mode algorithm is modified in order to design a velocity observer for systems with Coulomb friction. The finite time convergence of the observer is proved. The discrete realization of the observer is considered and the corresponding accuracy is estimated. The gain of the proposed observer is chosen taking into account only the terms with velocity, ignoring the elasticity terms. The usage of the proposed observer does not require the separation principle theorem, i.e. allows to design the controller and observer separately.

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